부 록

부록 A. 피셔정보행렬 (Fisher Information Matrix)의 유도

대수우도함수 $\ln L$을 $\delta_1, \delta_2, \beta$ 에 대해 일차 편미분하면 다음과 같다.

$$\frac{\partial \ln L}{\partial \delta_1} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left[A_i \ln \Delta y_{ijk} - A_i \ln \beta - A_i \Psi_0(A_i) \right] ,$$

$$\frac{\partial \ln L}{\partial \delta_2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left[A_i s_i \ln \Delta y_{ijk} - A_i s_i \ln \beta - A_i \Psi_0(A_i) s_i \right] ,$$

$$\frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left(-\frac{A_i}{\beta} + \frac{\Delta y_{ijk}}{\beta^2} \right)$$

여기서, $\Psi_0(A_i)$는 $\Psi_0(A_i) = \partial \ln \Gamma(A_i)/\partial A_i = \Gamma'(A_i)/\Gamma(A_i)$ 로 정의되는 디감마 함수 (Digamma function)이다. 대수우도함수 $\ln L$을 $\delta_1, \delta_2, \beta$ 에 대해 이차 편미분하면 다음과 같다.

$$\frac{\partial^2 \ln L}{\partial \delta_1^2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left[A_i \ln \Delta y_{ijk} - A_i \ln \beta - A_i \Psi_0(A_i) - A_i^2 B_i \right] ,$$

$$\frac{\partial^2 \ln L}{\partial \delta_1 \partial \delta_2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left[A_i s_i \ln \Delta y_{ijk} - A_i s_i \ln \beta - A_i \Psi_0(A_i) s_i - A_i^2 B_i s_i \right] ,$$

$$\frac{\partial^2 \ln L}{\partial \delta_1 \partial \beta} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left(-\frac{A_i}{\beta} \right) ,$$

$$\frac{\partial^2 \ln L}{\partial \delta_2^2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left[A_i s_i^2 \ln \Delta y_{ijk} - A_i s_i^2 \ln \beta - A_i \Psi_0(A_i) s_i^2 - A_i^2 B_i s_i^2 \right] ,$$

$$\frac{\partial^2 \ln L}{\partial \delta_2 \partial \beta} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left(-\frac{A_i s_i}{\beta} \right) ,$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left(\frac{A_i}{\beta^2} - \frac{2\Delta y_{ijk}}{\beta^3} \right)$$

피셔정보행렬을 산출하기 위한 $E(\ln \Delta y_{ijk})$ 은 다음과 같이 구할 수 있다.

$$
\begin{aligned}
E\left(\ln\Delta y_{ijk}\right) &= \int_0^\infty \ln\Delta y_{ijk}\,\frac{1}{\Gamma\left(A_i\right)}\frac{1}{\beta^{A_i}}\,\Delta y_{ijk}{}^{A_i-1}e^{-\Delta y_{ijk}/\beta}d\Delta y_{ijk}\\
&= \frac{1}{\Gamma\left(A_i\right)}\int_0^\infty \ln\left(\beta x\right)\cdot x^{A_i-1}e^{-x}dx\\
&= \frac{1}{\Gamma\left(A_i\right)}\left(\int_0^\infty \ln\beta\cdot x^{A_i-1}e^{-x}dx+\int_0^\infty \ln x\cdot x^{A_i-1}e^{-x}dx\right)\\
&= \frac{1}{\Gamma\left(A_i\right)}\left[\ln\beta\cdot\Gamma\left(A_i\right)+\Gamma'\left(A_i\right)\right]\\
&= \ln\beta+\Psi_0\left(A_i\right)
\end{aligned}
$$

그리고, $E(ly_{ijk})= A_i\beta$ 이므로 $\ln L$의 이차편미분의 음의 기댓값을 취하여 다음과 같고 식 (5)의 피셔정보 행렬을 구할 수 있다.

$$
E\left(-\frac{\partial^2\ln L}{\partial\delta_1^2}\right)=mN\sum_{i=1}^{r}\pi_i A_i^2 B_i \quad,\quad E\left(-\frac{\partial^2\ln L}{\partial\delta_1\partial\delta_2}\right)=mN\sum_{i=1}^{r}\pi_i A_i^2 B_i s_i \quad,
$$

$$
E\left(-\frac{\partial^2\ln L}{\partial\delta_1\partial\beta}\right)=mN\sum_{i=1}^{r}\pi_i\frac{A_i}{\beta} \quad,\quad E\left(-\frac{\partial^2\ln L}{\partial\delta_2^2}\right)=mN\sum_{i=1}^{r}\pi_i A_i^2 B_i s_i^2 \quad,
$$

$$
E\left(-\frac{\partial^2\ln L}{\partial\delta_2\partial\beta}\right)=mN\sum_{i=1}^{r}\pi_i\frac{A_i s_i}{\beta} \quad,\quad E\left(-\frac{\partial^2\ln L}{\partial\beta^2}\right)=mN\sum_{i=1}^{r}\pi_i\frac{A_i}{\beta^2} \quad.
$$